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Suspended Broadside-Coupled Slot Line with Overlay

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***Abstract*—This paper presents a rigorous analysis of symmetric, broadside-coupled slot line with overlay. The structure is assumed to be suspended inside a conducting enclosure of arbitrary dimensions. The dielectric substrate and the overlay are assumed to be isotropic and homogeneous and are of arbitrary thickness and relative permittivity. The conducting enclosure and the zero thickness metallization on the substrate are assumed to have infinite conductivity. The computed results illustrate a) the dispersion characteristics and characteristic impedance of the coupled slot line structure, b) the variation of the even-mode and also the odd-mode relative wavelength ratio and characteristic impedance with slot width, and c) the effect of shielding on the even-mode and also the odd-mode dispersion and characteristic impedance. This structure should find application in the design and fabrication of MIC components such as magic-T's and directional couplers.**

Manuscript received May 27, 1981; revised August 10, 1981.
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I. Introduction

THE PARALLEL-COUPLED slot line on a dielectric substrate is ideally suited for microwave integrated circuit components such as Magic-T's [1] and directional couplers [2], [3]. Recently, parallel-coupled slot line on double layer dielectric substrate and parallel-coupled slot line sandwiched between two dielectric substrates have been reported [4].

The paper presents an analysis of symmetric, broadside-coupled slot line with overlay and suspended inside a conducting enclosure of arbitrary dimensions. The dielectric substrate and the overlay are assumed to be isotropic

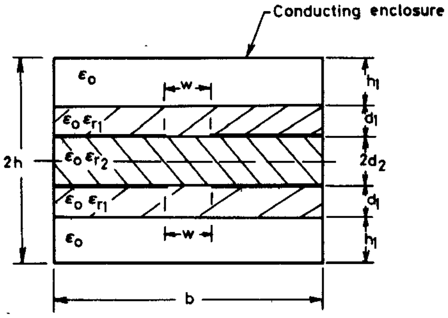

Fig. 1. Suspended broadside-coupled slot line with overlay.

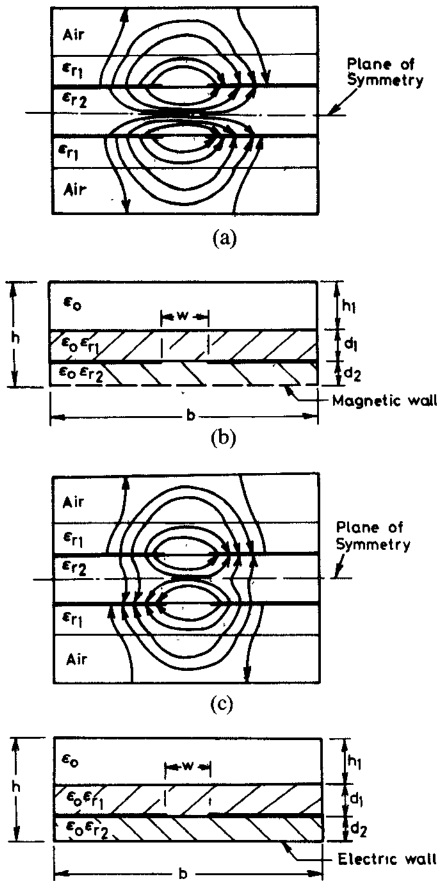

Fig. 2. (a) Even mode of excitation. (b) Upper half of the structure for the even mode of excitation. (c) Odd mode of excitation. (d) Upper half of the structure for the odd mode of excitation.

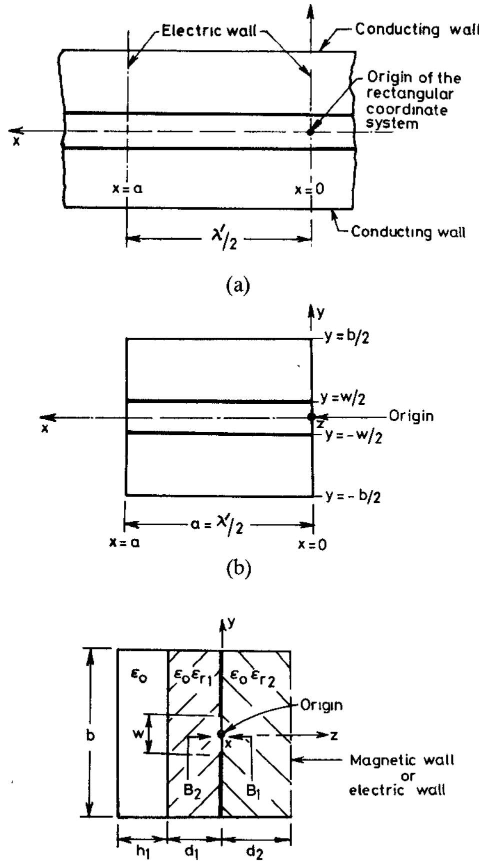

Fig. 3. (a) Insertion of transverse electric walls at $x=0$ and $x=a$. (b) Separation of the region having the rectangular waveguide boundary. (c) Waveguide model containing capacitive iris and dielectric substrates.

and homogeneous and are of arbitrary thickness and relative permittivity. The conducting enclosure and the zero thickness metallization on the substrate, are assumed to have infinite conductivity. The above structure is analyzed using Cohn's [5], [6] technique, which is extended here to take into account the presence of an identical slot on the reverse side of the substrate and also the effect of a shielding enclosure. For the modeling of the open structure the shielding enclosure is allowed to expand to infinity without causing numerical problems or increasing computing time.

II. Analysis

The structure to be analyzed is illustrated in Fig. 1. This structure is analyzed by taking into consideration the even mode and the odd mode of excitation, which are illustrated in Fig. 2(a) and (c). For the case of even excitation, a magnetic wall is placed along the plane of symmetry; it then suffices to restrict the analysis to the upper half of the structure (Fig. 2(b)). A similar simplification is possible for the case of odd excitation except that the magnetic wall at the plane of symmetry is replaced by an electric wall (Fig. 2(d)). In these structures slot waves of equal amplitude travelling in the $+x$ and $-x$ directions are taken into consideration. As a consequence, there exist along the slot line, transverse planes separated by $\lambda'/2$, where λ' is the wavelength in the slot line. At these planes the transverse electric field and the normal magnetic field get cancelled and become zero. Two such planes occur at $x=0$ and $x=\lambda'/2=a$; this is illustrated in Fig. 3 (a). Furthermore, electric walls can be inserted at these points without disturbing the field components between them. Then, the semi-infinite regions $x<0$ and $x>a$ can be eliminated. This is illustrated in Fig. 3(b). Thus the region that is separated from the original structure can be considered as a rectangular waveguide with a symmetric capacitive iris sandwiched between two dielectric substrates as illustrated in Fig. 3(c). The complete set of modes satisfying the boundary conditions of this structure are $TE_{1,2n}$ for n an integer $\geqslant 0$ and $TM_{1,2n}$ for n an integer $\geqslant 1$.

Since the wavelength in the slot line is less than the free space wavelength λ, it follows that a is less than $\lambda/2$. Therefore, the TE_{10} and all higher order modes are cutoff or nonpropagating in the air regions. In the dielectric regions the TE_{10} mode is propagating, and the first few higher modes may propagate or all the higher modes may be cut off depending upon the height of the waveguide b. When transverse resonance occurs the sum of the susceptances at the plane of the iris is zero. This sum includes the susceptance of the TE_{10} mode looking in the $-z$ and $+z$ directions, and the capacitive iris susceptance representing the $TE_{1,2n}(n>0)$ and $TM_{1,2n}(n>0)$ modes on the $-z$ and $+z$ sides of the iris. Let B_1 be the susceptance at the plane of the iris $(z=0)$ as seen to the left into the dielectric region of length d_1 terminated by an air region of length h_1 and B_2 be the susceptance at the plane of the iris $(z=0)$ as seen to the right into the dielectric region of length d_2 terminated in a magnetic or an electric wall. Then, the sum of the susceptance B_t, at the plane of the iris $(z=0)$ is B_1+B_2. To obtain a solution for $B_t=0$ an independent variable $p=\lambda/2a$ is defined. At the transverse resonance frequency $a=\lambda'/2$ and $p=\lambda/\lambda'$ for $B_t=0$. The wavelength and frequency for this solution are $\lambda=2a(\lambda/\lambda')$ and $f=c/\lambda$. An expression for the susceptance B_1 will be derived below. This is followed by an expression for the susceptance B_2 for the case of even excitation and also an expression for the susceptance B_2 for the case of odd excitation.

The total E_y and H_x fields at the $z=0$ plane and $x=a/2$ are functions of y as follows:

$$E_y=R_0+\sum_{n=1,2,\cdots}^{\infty} R_n \cos\frac{2\pi n y}{b} \tag{1}$$

and

$$H_x=-y_{i0}R_0-\sum_{n=1,2,\cdots}^{\infty} y_{in}R_n\cos\frac{2\pi n y}{b} \tag{2}$$

where the input wave admittances y_{i0} and y_{in} defined by

$$y_{i0}=-\left(\frac{H_x}{E_y}\right)_{TE_{10}} \tag{3}$$

and

$$y_{in}=-\left[\frac{(H_x)_{TE_{1,2n}}+(H_x)_{TM_{1,2n}}}{(E_y)_{TE_{1,2n}}+(E_y)_{TM_{1,2n}}}\right]. \tag{4}$$

The constants R_0 and R_n are determined as explained in [5], [7], and are as follows:

$$R_0=C_0\delta \qquad R_n=2C_0\delta\frac{\sin\pi n\delta}{\pi n\delta} \tag{5}$$

where $\delta=w/b$. Further, it can be shown that the input wave admittance as seen to the left at the $z=0$ plane is

$$Y_i=jB_1=Y_{i0}+2\sum_{n=1,2,\cdots}^{\infty} Y_{in}\left(\frac{\sin\pi n\delta}{\pi n\delta}\right)^2 \tag{6}$$

where Y_{i0} is the guide admittance seen by a TE_{10} mode wave directed into dielectric filled waveguide region of length d_1 terminated by an air-filled region of length h_1. Y_{i0} can be shown to be equal to

$$Y_{i0}=\frac{au_1}{2b}\tan\left[\frac{\pi u_1 d_1}{ap}-\tan^{-1}\frac{v}{u_1}\coth(\gamma h_1)\right] \tag{7}$$

where

$$u_1=(\epsilon_{r1}-p^2)^{1/2}$$
$$v=(p^2-1)^{1/2}$$
$$\gamma=vk. \tag{8}$$

γ is the TE_{10} mode propagation constant in the air region and $k=2\pi/\lambda$. Y_{in} is the guide admittance seen by the higher order TE and TM modes directed into the dielectric region. For each n, the corresponding TE and TM amplitudes must be so chosen so that E_x exactly cancels at $z=0$. In this way the total E_x field will be zero at $z=0$, as is required by the boundary condition in that transverse plane. When this condition is imposed, it can be shown that

$$Y_{in}=\frac{Y_{iTMn}+Y_{iTEn}(b/2an)^2}{1+(b/2an)^2} \tag{9}$$

and the input admittances Y_{iTMn} and Y_{iTEn} for each n are obtained from [7] by setting $\epsilon_{r2}=1$ or $d_2=0$ and substituting h_1 in place of h_2. Equations (6)–(9) yield the susceptance B_1 looking to the left at the plane $z=0$

$$jB_1=\frac{jau_1}{2b\eta}\tan\left[\frac{\pi u_1 d_1}{ap}-\tan^{-1}\frac{v}{u_1}\coth(\gamma h_1)\right]+2\sum_{n=1,2,\cdots}^{\infty} Y_{in}\left(\frac{\sin\pi n\delta}{\pi n\delta}\right)^2 \tag{10}$$

where $\eta=376.7\ \Omega$. However, the rate of convergence of this series is very slow and can be improved as explained in [5]. With the modification, (10), the susceptance B_1 looking to the left from the plane of the iris, is

$$\eta B_1=\frac{au_1}{2b}\tan\left\{\frac{\pi u_1 d_1}{ap}-\tan^{-1}\frac{v}{u_1}\coth(\gamma h_1)\right\}+\frac{u_1^2}{2p}\ln\left(\frac{2}{\pi\delta}\right)+\frac{1}{2p}\sum_{n=1,2,\cdots}^{\infty} M_{n1}\frac{\sin^2(n\pi\delta)}{n(n\pi\delta)^2}. \tag{11}$$

For F_{n1} real, M_{n1} is

$$M_{n1}=\frac{\epsilon_{r1}\tanh r_n-p^2F_{n1}^2\coth q_n}{\left[1+(b/2an)^2\right]F_{n1}}-u_1^2 \tag{12}$$

where

$$r_n = \frac{2\pi n F_{n1} d_1}{b} + \tanh^{-1}\left[\frac{F_{n1}}{F_n \epsilon_{r1}} \coth\left(\frac{2\pi n F_n h_1}{b}\right)\right] \quad (13)$$

$$q_n = \frac{2\pi n F_{n1} d_1}{b} + \coth^{-1}\left[\frac{F_n}{F_{n1}} \coth\left(\frac{2\pi n F_n h_1}{b}\right)\right] \quad (14)$$

$$F_n = \frac{b\gamma_n}{2\pi n} = \left[1 + \left(\frac{bv}{2anp}\right)^2\right]^{1/2} \quad (15)$$

and

$$F_{n1} = \frac{b\gamma_{n1}}{2\pi n} = \left[1 - \left(\frac{bu_1}{2anp}\right)^2\right]^{1/2}. \quad (16)$$

For F_{n1} imaginary, replace F_{n1} by $j|F_{n1}|$.

The susceptance B_2 looking to the right from the place of the iris for the even mode of excitation is derived in a similar manner as B_1 and is as follows:

$$\eta B_2 = \frac{au_2}{2b} \tan\left(\frac{\pi u_2 d_2}{ap}\right) + \frac{u_2^2}{2p} \ln\left(\frac{2}{\pi\delta}\right) + \frac{1}{2p} \sum_{n=1,2,\cdots}^{\infty} M_{n2} \frac{\sin^2(n\pi\delta)}{n(n\pi\delta)^2}. \quad (17)$$

where

$$u_2 = (\epsilon_{r2} - p^2)^{1/2}$$

and

$$F_{n2} = \frac{b\gamma_{n2}}{2\pi n} = \left[1 - \left(\frac{bu_2}{2anp}\right)^2\right]^{1/2}. \quad (18)$$

For F_{n2} real, M_{n2} is

$$M_{n2} = \frac{(\epsilon_{r2} - p^2 F_{n2}^2) \tanh(2\pi n F_{n2} d_2/b)}{\left[1 + (b/2an)^2\right] F_{n2}} - u_2^2. \quad (19)$$

For F_{n2} imaginary, replace F_{n2} by $j|F_{n2}|$.

When (11) and (17) are added, the total susceptances B_t at the plane of the iris for the even mode of excitation is obtained

$$\eta B_t = \frac{a}{2b}\left\{u_1 \tan\left[\frac{\pi u_1 d_1}{ap} - \tan^{-1}\frac{v}{u_1}\coth(\gamma h_1)\right] + u_2 \tan\left(\frac{\pi u_2 d_2}{ap}\right)\right\} + \frac{1}{p}\left\{\left[\frac{\epsilon_{r1}+\epsilon_{r2}}{2} - p^2\right]\ln\left(\frac{2}{\pi\delta}\right) + \frac{1}{2}\sum_{n=1,2,\cdots}^{\infty}(M_{n1}+M_{n2})\frac{\sin^2(n\pi\delta)}{n(n\pi\delta)^2}\right\}. \quad (20)$$

The susceptance B_2 looking to the right from the plane of the iris for the odd mode of excitation is as follows:

$$\eta B_2 = -\frac{au_2}{2b} \cot\left(\frac{\pi u_2 d_2}{ap}\right) + \frac{u_2^2}{2p} \ln\left(\frac{2}{\pi\delta}\right) + \frac{1}{2p} \sum_{n=1,2,\cdots}^{\infty} M_{n2} \frac{\sin^2(n\pi\delta)}{n(n\pi\delta)^2}. \quad (21)$$

For F_{n2} real, M_{n2} is

$$M_{n2} = \frac{(\epsilon_{r2} - p^2 F_{n2}^2) \coth(2\pi n F_{n2} d_2/b)}{\left[1 + (b/2an)^2\right] F_{n2}} - u_2^2. \quad (22)$$

For F_{n2} imaginary, replace F_{n2} by $j|F_{n2}|$.

When (11) and (21) are added, the total susceptance B_t at the plane of the iris for the odd mode of excitation is obtained

$$\eta B_t = \frac{a}{2b}\left\{u_1 \tan\left[\frac{\pi u_1 d_1}{ap} - \tan^{-1}\frac{v}{u_1}\coth(\gamma h_1)\right] - u_2 \cot\left(\frac{\pi u_2 d_2}{ap}\right)\right\} + \frac{1}{p}\left\{\left[\frac{\epsilon_{r1}+\epsilon_{r2}}{2} - p^2\right]\ln\left(\frac{2}{\pi\delta}\right) + \frac{1}{2}\sum_{n=1,2,\cdots}^{\infty}(M_{n1}+M_{n2})\frac{\sin^2(n\pi\delta)}{n(n\pi\delta)^2}\right\}. \quad (23)$$

III. Guide Wavelength and Characteristic Impedance

The values of ϵ_{r1}, ϵ_{r2}, d_1, d_2, w, b, h_1, and $a = \lambda'/2$ are substituted into (20) and (23) and solved for the value of p at which $\eta B_t = 0$. This p is equal to λ/λ'.

The ratio of phase velocity to group velocity v/v_g and the slot-wave characteristic impedance Z_0 are defined as follows [5]:

$$\frac{v}{v_g} = 1 + \frac{f}{\lambda/\lambda'} \frac{\Delta(\lambda/\lambda')}{\Delta f} \quad (24)$$

$$Z_0 = 376.7 \frac{v}{v_g} \frac{\pi}{p} \frac{\Delta p}{-\Delta(\eta B_t)}, \quad \Omega. \quad (25)$$

v/v_g and Z_0 are computed as explained in [5], [7].

IV. Numerical Results

The even-mode and also the odd-mode dispersion characteristics and characteristic impedance of the coupled slot-line structure are illustrated in Fig. 4. It is observed that the ratio of the phase velocities is close to unity and also the even-mode characteristic impedance is greater than the odd-mode characteristic impedance. Fig. 5 illustrates the variation of the even-mode and odd-mode relative wavelength ratio and also the characteristic impedance as a function of the slot width. The height and width of the conducting enclosure is held fixed. It is observed that the even-mode characteristic impedance is more sensitive than the odd-mode characteristic impedance to the slot-width variation. Finally, the effect of varying the distance of separation between the top and bottom shielding covers

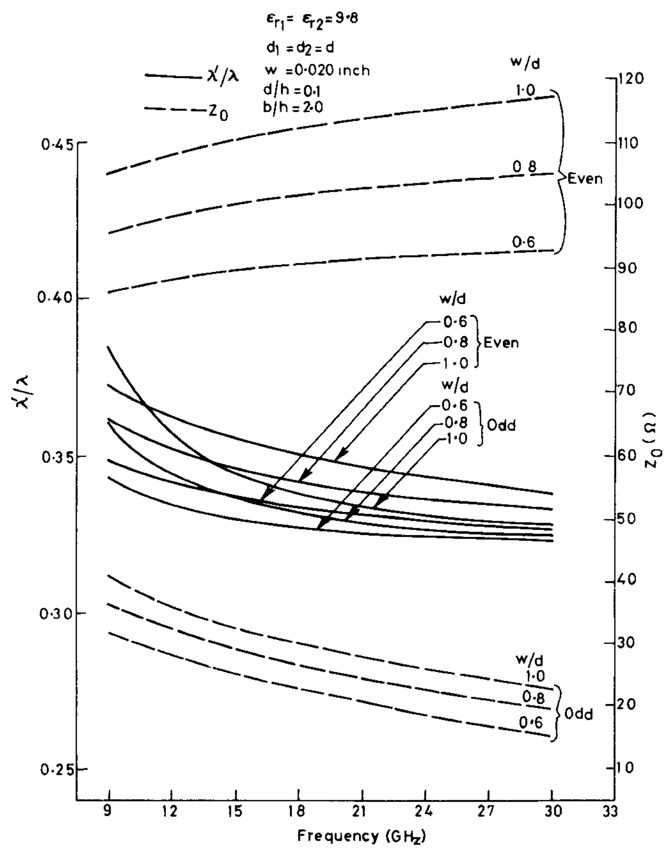

Fig. 4. The dispersion characteristics and characteristic impedance of the coupled slot-line structure.

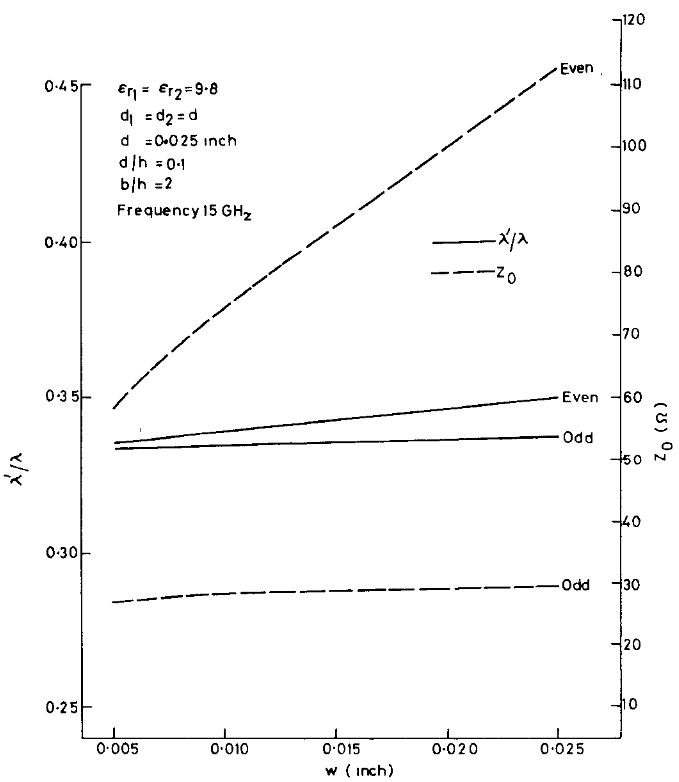

Fig. 5. The even-mode and the odd-mode relative wavelength ratio and characteristic impedance versus the slot width.

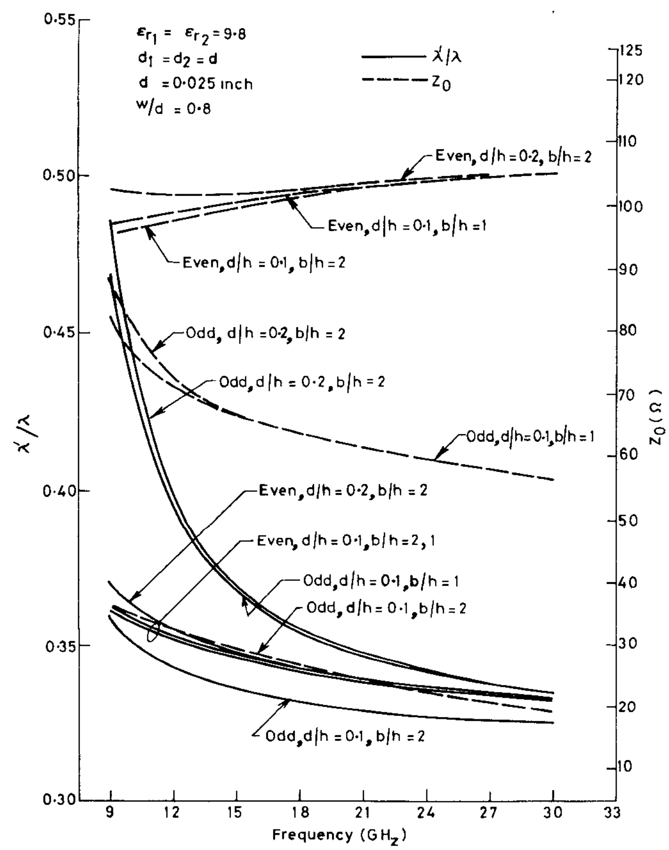

Fig. 6. The effect of shielding on the even-mode and the odd-mode dispersion and modal impedances.

and also the effect of lateral shielding on the even-mode and odd-mode dispersion and the modal impedances are illustrated in Fig. 6. It is observed that the odd-mode dispersion and characteristic impedance is more sensitive than the even-mode dispersion and characteristic impedance to the variation in the height and width of the shielding enclosure.

V. Conclusion

Briefly, the paper presents an analysis of suspended broadside-coupled slot line with overlay. An expression for the total susceptance at the plane of the iris is obtained for the even and the odd modes of excitation. The dispersion characteristics and characteristic impedance are computed and graphically illustrated. Since the ratio of the two phase velocities is close to unity, it should be possible to design directional couplers having high directivitiy using broadside-coupled slot line with overlay. The effect of shielding and also the effect of varying the slot width on the dispersion and characteristic impedance are graphically illustrated.

These results should find extensive application in the design of microwave and millimeter wave integrated circuits components, such as magic-T's, directional couplers, and filters.

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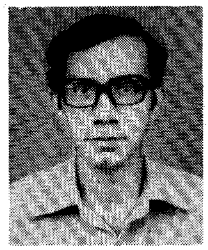

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